

MECO In An Exponential Metric

Stanley L. Robertson ¹

Abstract

Magnetic Eternally Collapsing Objects (MECO) have been proposed as the central engines of galactic black hole candidates (GBHC) and supermassive active galactic nuclei (AGN). Previous work has shown that their luminosities and spectral and timing characteristics are in good agreement with observations. These features and the formation of jets are generated primarily by the interactions of accretion disks with an intrinsically magnetic central MECO. The interaction of accretion disks with the anchored magnetic fields of the central objects permits a unified description of properties for GBHC, AGN, neutron stars in low mass x-ray binaries and dwarf novae systems. The previously published MECO models have been based on a quasistatic Schwarzschild metric of General Relativity; however, the only essential feature of this metric is its ability to produce extreme gravitational redshifts. For reasons discussed in this article, an alternative development based on a quasistatic exponential metric is considered here.

1 Introduction

Stellar mass objects compact enough to be black holes are ubiquitous in galaxies while supermassive compact objects are found in the nuclei of most, if not all, galaxies. Although these objects are routinely called black holes, no compelling evidence of an event horizon, the quintessential feature of a black hole, has yet been found. Though not widely considered, it is possible that these objects may not possess event horizons at all. They might instead be some kind of eternally collapsing object (ECO) that exhibits extreme gravitational redshift and consequently, very low luminosity.

Once an object becomes smaller than its photon sphere, outgoing radiation begins to be trapped; the more compact it becomes, the more efficient the trap. Within the scope of general relativity, it has been shown (Herrera & Santos 2004, Herrera, DiPrisco & Barreto 2006, Mitra 2006a,b,c, Mitra & Glendenning 2010, Robertson & Leiter 2004) that it is possible to achieve an Eddington balance before the formation of trapped surfaces or event horizons. There is evidence that such non-black hole objects exist and that they are strongly magnetic (Robertson & Leiter 2002, 2003, 2004 hereafter RL02, RL03, RL04, Schild, Leiter & Robertson 2006, 2008, Schild & Leiter 2010) The Robertson-Leiter MECO variant adds a (M)agnetic field that can reach a quantum electrodynamic limit. It contributes heavily to the outgoing photon flow (Robertson & Leiter 2006, 2010 hereafter RL06, RL10). This eventually allows all of the observable properties of their MECO model to depend only

¹Dept. of Chemistry & Physics, Emeritus, Southwestern Oklahoma State University
Weatherford, OK 73096, (stan.robertson@swosu.edu)

upon two parameters - mass and spin, but MECO models with other magnetic field configurations might also be of interest. Despite the high surface luminosity of an Eddington balance, distantly observed MECO are very low luminosity objects capable of satisfying all observational constraints due to extreme gravitational redshift of the photons that do escape.

There are strong observational signatures of the magnetic central objects within Low Mass X-ray Binary systems, AGN and dwarf novae. Low and high luminosity states, spectral state switches between them, quasi-periodic oscillations and jet formation are common features. These states and oscillations have been described in terms of the interaction of accretion disks threaded by poloidal magnetic fields (Chou & Tajima 1999). Some very revealing simulations of some of the process can be seen here

<http://astrosun2.astro.cornell.edu/us-rus/jets.htm> along with interesting animations such as this one

http://astrosun2.astro.cornell.edu/us-rus/animation/a_funnel.htm which can be compared with observational data and correlations of quasi-periodic oscillations of dwarf novae, neutron stars and GBHC (Mauche 2002, Warner & Woudt 2003)

In previous work (RL02, RL04, RL06) evidence was provided for the existence of intrinsic magnetic moments of $\sim 10^{29-30} \text{ Gauss cm}^3$ in the GBHC of LMXB. Others have reported evidence for strong magnetic fields in GBHC. A field in excess of 10^8 G has been found at the base of the jets of GRS 1915+105 (Gliozzi, Bodo & Ghisellini 1999, Vadawale, Rao & Chakrabarti 2001). Based on a study of optical polarization of Cygnus X-1 in its low state (Gnedin et al. 2003) has found a slow GBHC spin and a magnetic field of $\sim 10^8 \text{ Gauss}$ at the location of its optical emission. Since these field strengths exceed disk plasma equipartition levels they can be attributed to a magnetic central object. They are in good agreement with the magnetic moments that have previously been presented (RL06) It was also shown (RL04) that the MECO model and its jet mechanism scales up to the AGN without difficulty. The empty inner disk region shown by microlensing of quasars (Schild, Leiter & Robertson 2006, 2008, Schild & Leiter 2010) has been shown to be consistent with the magnetic field structure of the MECO model. Lastly, it was shown that the MECO model can reconcile the low luminosity of Sgr A* with its expected Bondi accretion rate and account for the orthogonal polarizations of near infrared and millimeter radio emissions of Sgr A* (RL10).

The most attractive feature of the MECO model is that a central object with an intrinsic magnetic field interacts with accretion disks in a way that permits a unified description of the similar observable characteristics of dwarf novae, LMXB neutron stars, GBHC and AGN. While there are some observable differences between neutron stars, dwarf novae and GBHC, it has not been demonstrated that any of the differences can be attributed to the presence of an event horizon. The only thing that is needed is a large gravitational redshift to satisfy the requirements of

low luminosity. The success of the MECO models demonstrates clearly that there is as yet no need of event horizons in astrophysics. That being the case, it is of some interest to consider alternatives that do not possess them. The essential ingredients and details of MECO are recapitulated here for an exponential metric that is capable of producing the required redshifts.

But with a successful model of MECO based on a Schwarzschild metric one might ask why a different basis for MECO might be of interest. The answer is that we hope to eventually settle the question of whether or not black holes exist by comparing observations and models with and without event horizons. A model based on a metric with no event horizon might help to clarify the issues. Stated another way, before the existence of black holes is accepted it ought to be necessary to find black hole candidates exhibiting characteristics that cannot be easily accommodated by a metric that lacks an event horizon.

2 The Exponential Metric

Event horizons are problematic in their own right. The question of how quantum mechanics and general relativity might be reconciled has recently been sharpened by considering what happens to a freely falling particle of matter approaching an event horizon. The possibility that it might meet a radiative “firewall” has recently become a very active research topic (e.g., Abramowicz, Kluzniak & Lasota 2013, Anastopoulos & Savvidou, 2014, Hawking 2014). Hawking says that event horizons cannot exist as stable features of compact objects.

This is a problem of such importance that we should consider all aspects; however, the necessity of event horizons in astrophysics seems not to have been questioned. They first arose in the solutions of the Einstein field equations applicable to the exterior of a central compact mass and are manifest by the vanishing of the time component of the metric tensor, which for the Schwarzschild metric, is

$$g_{00} = 1 - 2R_g/r, \quad R_g = GM/c^2 \quad (1)$$

The gravitational redshift z , of photons leaving the surface of a Schwarzschild metric compact object is given by $1 + z = g_{00}^{-1/2}$, which is singular with $g_{00} = 0$ at the Schwarzschild radius, $R_s = 2R_g$. This occurrence of an event horizon can be traced to Einstein’s exclusion of gravitational field energy as a source of spacetime curvature. All forms of mass-energy EXCEPT that of gravitational fields were considered as sources. Nevertheless, the (dimensionless) Newtonian gravitational potential function $u(r) = GM/(c^2/r)$ is actually a solution of the Einstein field equations if the gravitational field energy given by $(c\nabla u(r))^2/(8\pi G)$ is used as a source term in the space exterior to mass M in the right member of the Einstein field equations (see Appendix B). The field energy is of second degree in the derivative of the potential and is of second order in its affect on curvature. If it is truly needed for a correct description of gravity, then General Relativity would seem to be just a

first order theory; good enough for weak fields but not to be trusted in the strong fields of objects as compact as $2R_g$.

If the gravitational field is a physical entity in its own right, rather than being manifest solely by a spacetime curvature that has excluded the effect of gravitational field energy, then one might consider gravitational redshift to be a function of the (dimensionless) gravitational potential $u(r) = GM/(c^2 r)$.

Consider the frequencies of photons leaving the surface of a compact mass, M , and being observed at locations a, b and c in the external space. If the corresponding observed frequencies would be ν_a, ν_b, ν_c , then their ratios would be functions of the potentials such as $\nu_a/\nu_b = f(u(r_a), u(r_b))$. But since potentials are only determined to within an additive constant, this would be the same as $f(u(r_a) + C, u(r_b) + C)$. So consider $\nu_a/\nu_b = (\nu_a/\nu_c)(\nu_c/\nu_b) = f(u(r_a) + C, u(r_b) + C) = f(u(r_a), u(r_c))f(u(r_c), u(r_b))$. If we would choose $C = -u(r_b)$ then the first ratio would become $f(u(r_a) - u(r_b), 0)$. Similarly we could add a constant $-u(r_c)$ to each of the next two terms and obtain $f(u(r_a) - u(r_b), 0) = f(u(r_a) - u(r_c))f(0, u(r_b) - u(r_c))$. If we let $A = u(r_a) - u(r_c)$ and $B = u(r_c) - u(r_b)$ we have $f(A + B, 0) = f(A, 0)f(0, -B)$. But since it must also be possible to offset the potentials by a constant amount $+B$, it follows that $f(0, -B) = f(B, 0)$. So we arrive at $f(A + B, 0) = f(A, 0)f(B, 0)$. Thus we conclude that the observed frequencies of photons should be proportional to an exponential function, $f(u(r)) = e^{u(r)}$, and the gravitational redshift observed where $u(r) = 0$ should be given by

$$1 + z = e^{u(r)} = e^{R_g/r} \quad (2)$$

It is of considerable interest to note that this result also follows exactly from Einstein's principle of equivalence and the special theory of relativity (see Appendix A).

If we consider a meter stick transported from the compact object surface out to various distances to be dependent on the gravitational potential, we would conclude that its length must also be an exponential function of the potential. If the orientation of the stick relative to the gravitational field direction would not matter, then we can describe the effects of gravity in terms of the isotropic metric form:

$$ds^2 = e^{-2u(r)} c^2 dt^2 - e^{2u(r)} (dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2) \quad (3)$$

This is the generic metric that will be used for the remainder of this article. It was first presented in this way by Rastall (1975). While this metric is far, far from a complete gravity theory it can encompass essentially all of relativistic astrophysics except gravitational radiation and waves and large scale cosmology. Its main appeal in the present case is that it can yield large and arguably correct gravitational redshifts without an event horizon.

Several features of immediate interest pertain to geodesic motions of particles in this exponential metric: (i) There is an innermost marginally stable orbit at a distance of $5.24R_g$ from the central object center, which is only slightly smaller than

the $6R_g$ of the Schwarzschild metric. This small difference is due to the isotropic form of the metric. (ii) The energy that can be liberated in an accretion disk that reaches the marginally stable orbit is 5.5% of the rest mass energy of the particles, compared to 5.7% in the Schwarzschild metric. (iii) There is a circular photon orbit and “photon sphere” at $r = 2R_g$, compared to $3R_g$ in the Schwarzschild metric. There are no stable closed orbits that exist within or pass inside the photon sphere for particles with nonzero rest mass, however, particles with sufficient energy might escape from inside the photon sphere. (iv) Unlike a Schwarzschild metric, there is no event horizon from which photons cannot escape. Photons with sufficiently radial motions can always escape, however those not directed radially are increasingly restricted to small escape cones. It can be easily shown that escape geodesics of photons in the exponential metric are restricted to maximum angles of departure from the vertical that are proportional to e^{-2u}/r . Since this has a maximum for $u = 1/2$, the fraction of photons emitted isotropically from within $r < 2R_g$ that can escape is

$$f_{esc} = [2u(r)e^{(1-2u(r))}]^2 \quad (4)$$

3 MECO in the Exponential Metric

The general structure of the MECO is that of a strongly radiation pressure dominated plasma with a core temperature that might be as large as 10^{13}K (Mitra 2006a) in GBHC. As radiation streams outward and cools, an Eddington limit is reached at which there is no longer sufficient outgoing radiation flux to support baryons. This is taken to be the MECO “surface”. The compactness there then guarantees (Cavaliere & Morrison 1980, see Appendix D) that photon-photon collisions would copiously produce electron-positron pairs in the surface region. This should have the effect of buffering the temperature at about the pair production threshold. This makes the baryon “surface” a phase transition zone at the base of an electron-positron pair atmosphere. As temperature declines further out in the pair atmosphere the concentration of pairs decreases. Eventually a photosphere is reached. This is taken to be a last scattering surface, but the MECO is still so compact that only a small fraction of isotropically emitted photons actually escape from the photosphere.

3.1 Central MECO Quiescent Luminosity

Many, if not most GBHC, LMXB and AGN produce their transient high luminosities from accretion disks. For the moment, however, the luminosity of the MECO engine is the subject of interest. Throughout the MECO object interior, a plasma with some baryonic content is supported by a *net* outward flux of momentum via radiation at

the local Eddington limit L_{Edd} . At the baryon surface this is given by

$$L_{Edd,s} = \frac{4\pi G M c (1 + z_s)}{\kappa} \quad (5)$$

Here κ is the opacity of the plasma, subscript s refers to the baryonic surface layer and z_s is the gravitational redshift at the surface given by Eq. 2. For a hydrogen plasma, $\kappa = 0.4 \text{ cm}^2 \text{ g}^{-1}$ and

$$L_{Edd,s} = 1.26 \times 10^{38} m (1 + z_s) \text{ erg s}^{-1} \quad (6)$$

where $m = M/M_\odot$ is the mass in solar mass units.

Beyond the baryon surface, the pair atmosphere remains opaque. The net outward momentum flux continues onward, but diminished by two effects, time dilation of the rate of photon flow and gravitational redshift of the photons. The escaping luminosity at a location where the redshift is z is thus reduced by the ratio $(1 + z)^2 / (1 + z_s)^2$, and the net outflow of luminosity as radiation transits the pair atmosphere and beyond is

$$L_{net \text{ out}} = \frac{4\pi G M c (1 + z)^2}{\kappa (1 + z_s)} \quad (7)$$

Finally, as distantly observed where $z \rightarrow 0$, the luminosity is

$$L_\infty = \frac{4\pi G M c}{\kappa (1 + z_s)} \quad (8)$$

For hydrogen plasma opacity of $0.4 \text{ cm}^2 \text{ g}^{-1}$, and a typical GBHC mass of $7 M_\odot$ this equation yields $L_\infty = 8.8 \times 10^{38} / (1 + z_s) \text{ erg s}^{-1}$. But since the quiescent luminosity of GBHC are observed to be less than about $10^{31} \text{ erg s}^{-1}$, it is necessary to have $z_s \approx 10^8$. Even larger redshifts are needed to satisfy the quiescent luminosity constraints for AGN. This is extraordinary, to say the least, but no more incredible than the $z = \infty$ of a black hole. Achieving a redshift of 10^8 in the exponential metric would require $u(R) \sim 18$ and $R = R_g/18$. This would, indeed, be a very compact object.

At the low luminosity of Eq. 8, the gravitational collapse is characterized by an extremely long radiative lifetime, τ (RL03, Mitra 2006a) given by:

$$\tau = \frac{\kappa c (1 + z_s)}{4\pi G} = 4.5 \times 10^8 (1 + z_s) \text{ yr} \quad (9)$$

With the large redshifts that would be necessary for consistency with quiescent luminosity levels of BHC, it is clear why such a slowly collapsing object would be called an “eternally collapsing object” or ECO.

At the outskirts of the pair atmosphere of an ECO the photosphere is reached. Here the temperature and density of pairs has dropped to a level from which photons

can depart without further scattering from positrons or electrons. Nevertheless, the redshift is still large enough that their escape cone is small and most isotropically emitted photons will not travel far before falling back through the photosphere. Let the photosphere radius, temperature and redshift be R_p , T_p and z_p , respectively, and note that the proper radiating area at the photosphere in the isotropic metric is $4\pi R_p^2 e^{2u(R_p)}$ and $u(R_p) = R_g/R_p$. The net escaping luminosity is

$$L = 4(R_g/R_p)^2 e^{2(1-2u(R_p))} 4\pi e^{2u(R_p)} R_p^2 \sigma T_p^4 = \frac{4\pi G M c (1+z_p)^2}{\kappa(1+z_s)} \quad (10)$$

But in the pure radiation regime beyond the photosphere, the temperature and redshift are related by

$$T_\infty = \frac{T}{1+z} \quad (11)$$

where T_∞ is the distantly observed radiation temperature. Substituting $1+z_p = e^{u(R_p)}$ into Eq. 10 and solving for T_∞ , yields

$$T_\infty = \frac{2 \times 10^6}{[m(1+z_s)]^{1/4}} \quad K \quad (12)$$

Temperatures given by Eq. 12 are lower by a factor of 12 than those previously obtained (RL04, RL06) for a Schwarzschild metric MECO. For example, as distantly observed, $T_\infty \sim 10^4 \text{ K}$ for a $10M_\odot$ GBHC. When the radiation that escapes the photosphere is observed at a large distance, R , where $z = 0$, the observed fluence would be

$$\frac{L_\infty}{4\pi R^2} = 4e^2 (R_g)^2 \sigma T_\infty^4 \quad (13)$$

The right member of Eq. 13 can be written in terms of the distantly observed spectral distribution, for which the radiant flux density at distance R ($> 2R_g$) and frequency ν_∞ would be

$$F_{\nu_\infty} = \frac{2\pi h \nu_\infty^3}{c^2} \frac{1}{e^{(h\nu_\infty/kT_\infty)} - 1} \frac{4e^2 R_g^2}{R^2} \quad (14)$$

With a factor of $4e^2$ taking the place of the number 27 found for the Schwarzschild metric, this result is hardly changed from what was obtained previously for a Schwarzschild metric MECO. Previous application of the equation (RL10) for the quiescent emissions of Sgr A* showed that the MECO model satisfied the observational maximum luminosity constraints (Broderick, Loeb & Narayan 2009). With the lower temperatures given by Eq. 12, the maximum luminosity calculated from Eq. 14 is reduced even further.

3.2 Redshift and Magnetic Field

It seems likely that GBHC are formed via stellar core collapse. One would expect the collapsing plasma to produce magnetic fields via flux compression comparable to the $\sim 10^{13-14}$ G found in young neutron stars. The interior magnetic field near the MECO baryon surface is assumed to be of this magnitude. The exterior magnetic field near the base of the pair atmosphere is much larger due to the presence of drift currents. In contrast with neutron stars, where a relatively non-conducting crust might allow continuity of tangential magnetic field components across the boundary, the surface region of a MECO would be highly conductive. Plasma in the vicinity of the surface would be subject to drift currents proportional to $\mathbf{g} \times \mathbf{B}/B^2$. It then follows from Ampere's law that the interior and exterior fields at the surface must differ.

Distantly observed dipole fields that would originate on a redshifted surface have been previously examined (Baumgarte & Shapiro 2003, Rezzola et al. 2001). Denoting the tangential components of magnetic field above and below the surface, S, as $B_{\theta,S+}$ and $B_{\theta,S-}$, their ratio was shown to be (RL06)

$$\frac{B_{\theta,S+}}{B_{\theta,S-}} = \frac{3(1+z_s)}{6\ln(1+z_s)} \quad (15)$$

As noted above, a redshift of $\sim 10^8$ would be needed in order to satisfy the luminosity constraints for GBHC. But if $B_{\theta,S-} \sim 10^{12-14}$ G, then at the surface the field strength given by Eq. 15 could be as large as 10^{20} G. In fact, it cannot be much larger than the quantum electrodynamically determined maximum value for a NS given by $B_{\theta,S} \sim 10^{20}$ G (Harding, A., 2003). This is because surface magnetic fields much larger than $\sim 10^{20}$ G would create a spontaneous quantum electrodynamic phase transition associated with the vacuum production of bound pairs on the MECO surface (Zaumen 1976). For this reason, Robertson & Leiter estimated $B_{\theta,S+} \sim 10^{20}$ G and estimated $B_{\theta,S-} \sim 10^{13.4}$ G (Gupta, Mishra, Mishra & Prasanna 1998). For MECO objects differing in mass from GBHC, this last value needs to be modified by appropriate scaling. Even if the exterior field strength is not sufficient to produce bound pairs on the MECO surface the temperature there will be above the pair production threshold and there will be a pair plasma in the surface region. At the low baryon density near the surface, the interior field should be essentially an equipartition field. Deeper within the MECO, with densities approaching nuclear density, the field would remain well below equipartition. At equipartition the photon pressure in a pair plasma is proportional to B^4 (Pelletier & Marcowith 1998). It must be capable of countering the gravitational pressure, which is proportional to the mass density. At the compactness of a MECO, the mass density scales as m^{-2} . Thus it is necessary to require $B^4 \propto m^{-2}$ or $B \propto m^{-1/2}$. Incorporating this scaling and starting from a redshift appropriate for a GBHC of $\sim 7M_\odot$, Eq. 15 can be solved iteratively to yield

$$1+z_s = 5.7 \times 10^7 \sqrt{m} \quad (16)$$

where $m = M/M_\odot$, as usual. The magnetic moment corresponding to a 10^{20} G surface magnetic field would be

$$\mu \sim 1.5 \times 10^{28} m^{5/2} \text{ G cm}^3 \quad (17)$$

This magnetic moment, and the spin rates given by Table 1, Eq. 6 give the MECO model a good correspondence with observations of spectral state switches and the radio luminosities of jets for both GBHC and AGN (RL03, RL04, RL06).

The combination of an equipartition field at the MECO surface and a strong redshift dependence provides a measure of stability for the MECO. If the field increased for some reason, this would cause more pairs to be produced than just those required for the Eddington balance of the MECO-GBHC surface region. Additional photon pressure would then cause the MECO-GBHC surface zone to expand. However the resultant expansion due to this process would reduce the redshift and the magnetic field thus quenching the vacuum production of bound pairs and allowing the MECO-GBHC surface to contract. Similarly, if the surface zone contracted excessively, the increase of redshift would lead to a stronger field and more pair production, hence more radiation pressure to support the surface zone.

Finally, note that in both GBHC and AGN, the maximum luminosity of the low-hard x-ray states and a cutoff of radio emissions from jets occurs at about 2% of Eddington luminosity. To accommodate these facts, it is necessary for the magnetic moment to scale as $m^{5/2}$, as it does in the MECO model.

3.3 The Photosphere

The decline of temperature within the pair atmosphere will eventually lead to a decrease in pair density and the occurrence of a photosphere as a last scattering surface for outbound photons. The escape cone at the photosphere is so small, however, that few photons escape from there even without further scattering. The photosphere can be found from the condition that (Kippenhahn & Wiggert 1990)

$$\int_{\infty}^{R_p} n_{\pm} \sigma_T dl = 2/3 \quad (18)$$

where dl is an increment of proper length in the pair atmosphere, n_{\pm} is the combined number density of electrons and positrons along the path, and σ_T is the Thompson photon-electron collision cross-section. Landau & Lifshitz (1958) show that

$$n_{\pm} = \frac{8\pi}{h^3} \int_0^{\infty} \frac{p^2 dp}{\exp(E/kT) + 1} \quad (19)$$

where p is the momentum of a particle, $E = \sqrt{p^2 c^2 + m_e^2 c^4}$, k is Boltzmann's constant, h is Planck's constant and m_e , the mass of an electron. For low temperatures such that $kT < m_e c^2$ this becomes:

$$n_{\pm} \approx 2 \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} \exp(-m_e c^2 / kT) \quad (20)$$

$$= 2.25 \times 10^{30} (T_9)^{3/2} \exp(-1/T_9) \text{ cm}^{-3}$$

where $T_9 = T/(6 \times 10^9) \text{ K}$.

The path increment, dl , can be written in terms of changing redshift as

$$dl = dr(1+z) = e^{u(r)} dr \quad (21)$$

Substituting into Eq. 18 and using $r = GM/(c^2 u(r))$, $1+z = e^{u(r)}$ and $T = T_\infty(1+z)$ beyond the photosphere provides the relation

$$\frac{2.22 \times 10^{11} m}{T_{\infty,9}} \int_{T_{\infty,9}+\epsilon}^{T_p} \frac{T_9^{3/2} e^{(-1/T_9)} dT_9}{(\ln(T_9/T_{\infty,9}))^2} = 2/3 \quad (22)$$

Here ϵ was taken as 10^{-6} just to avoid starting with a divergent logarithm term at $T = T_\infty$. In actual fact, the pair density is negligibly small for temperatures still well in excess of $T = T_\infty$. Using Eqs. 11 and 16, Eq. 22 has been numerically integrated to obtain the photosphere temperatures and redshifts for various masses. The results are represented with errors below 1% for $1 < m < 10^{10}$ solar mass by the relations:

$$T_p = 2.5 \times 10^8 m^{-0.034} \quad K \quad (23)$$

and

$$1 + z_p = 1.1 \times 10^4 m^{0.343} \quad (24)$$

This is a much larger redshift than was obtained for the Schwarzschild metric, but the photosphere temperature here is about half as large.

3.4 Central MECO Luminosity Under Accretion Conditions

It is true that a MECO will eventually achieve 100% efficiency of conversion of accretion mass-energy to outgoing radiation, but the conversion takes place on the time scale of the extremely long MECO radiative lifetime. Accreting particles that reach the photosphere do not produce a hard radiative impact. They first encounter soft photons, then harder photons, then electron-positron pairs at the photosphere and they eventually reach the baryon surface where the net outflowing luminosity is already at the local Eddington limit rate. This provides a very soft landing in a phase transition zone among the baryons.

The adjustment to additional mass reaching the baryon zone of the MECO would take place on a local acoustic wave proper time scale of order $GM/c^3 \sim 5m \times 10^{-6}$ s, which is about ten orders of magnitude less than the time required for photons to diffuse from the baryon surface through the photosphere (RL10). Thus the MECO has ample time to adjust to any additional accreted mass that it acquires without requiring that it be immediately radiated away. On the other hand, the time for accreted baryons to diffuse on into the interior is also long. At high accretion rates they might well contribute significantly to the pressure in the surface layers.

This should be matched by an increase of radiation pressure and additional pair generation. Because of the high redshift, it is safe to say that none of the rapid variability of GBHC or AGN would originate from a MECO baryon surface or photosphere, but it might be possible that accretion would result in an increase of luminosity flowing outward.

The only thing that is required of the MECO to maintain its stability is for the radiation pressure in the accretion zone to increase by enough to counter the pressure supplied by accretion. This requirement can be examined by considering the pressure that accreting matter might exert if stopped at the baryon surface. The speed of a particle of rest mass m_o in radial free fall would be moving at essentially the speed of light when it reaches the surface. According to an observer at the surface, it would have a momentum of $\gamma m_o c$, where $\gamma = 1 + z_s$ is the Lorentz factor². If particles arrive at the locally observed rate of dN/dt_s , then the quantity of momentum deposited according to the observer at the surface would be $(dN/dt_s)(1 + z_s)m_o c$. With the substitution of $dt/(1 + z_s)$ for dt_s , this becomes $(dN/dt)m_o(1 + z_s)^2 c$. Thus the rate of momentum transport to the surface as observed there would be $\dot{m}_\infty(1 + z_s)^2 c$, where $\dot{m}_\infty = m_o dN/dt$ is the mass accretion rate as determined by a distant observer. If this momentum were deposited uniformly at the surface it would generate accretion pressure $p_{accr} = \dot{m}_\infty(1 + z_s)^2 c / (4\pi e^{2u(R_s)} R_s^2) = \dot{m}_\infty c / (4\pi R_s^2) = \dot{m}_\infty c [\ln(1 + z_s)]^2 / (4\pi R_g^2) = 1.6 \times 10^4 \dot{m}_\infty [\ln(1 + z_s)]^2 / m \text{ erg cm}^{-3}$.

Worst cases for this can be examined for \dot{m}_∞ large enough to make the luminosity of an accretion disk reach the level of an Eddington limit for the MECO as judged by a distant observer. This rate would be $\dot{m}_\infty/m = 1.38 \times 10^{38} / (0.055c^2) = 2.8 \times 10^{18} \text{ s}^{-1}$. This would provide an accretion pressure of $p_{accr} \sim 10^{25} \text{ erg cm}^{-3}$. This is roughly three times larger than the $3 \times 10^{24} \text{ erg cm}^{-3}$ radiation pressure that would exist with a surface temperature of $6 \times 10^9 \text{ K}$, but all that it would do would be to raise the MECO luminosity above its quiescent level by about the same factor. It simply would not be noticeable in the face of a much brighter external accretion flow. For the much lower accretion rate found for Sgr A*, it was shown (RL10) that MECO accretion luminosity was well below the observational upper limits. Thus the claim that its low luminosity is proof of the existence of an event horizon (Broderick, Loeb & Narayan 2009) is false.

4 MECO - Accretion Disk Interactions

A large number of GBHC have been found in LMXB systems. These consist of compact objects such as neutron stars or black hole candidates paired with dwarf stars that contribute mass that episodically flows into the compact objects and produces x-ray nova outbursts. These nova systems are well explained by a disk

²From the geodesic equations of motion, the distant coordinate time, t , and proper time, τ , moving with the particle are related by $dt = (1 + z)^2 d\tau$. An interval dt_s for an observer at rest at the surface is related by $dt_s = \gamma d\tau = dt/(1 + z_s)$, from which it follows that $\gamma = 1 + z_s$

instability model. Mass tends to accumulate in large radius accretion disks until its own viscosity heats it to the point of ionization. That triggers a rapid inflow that is collimated by the gravitational field of the compact companion into an accretion disk. The disk fills on a viscous timescale and the x-ray luminosity goes through a series of characteristic spectral states as the disk engages the magnetic field of the central object.

Standard gas pressure dominated ‘alpha’ accretion disks would be compatible with MECO. In LMXB, when the inner disk engages the magnetosphere, the inner disk temperature is generally high enough to produce a very diamagnetic plasma. Surface currents on the inner disk distort the magnetopause and they also substantially shield the outer disk such that the region of strong disk-magnetosphere interaction is mostly confined to a ring or torus. This shielding leaves most of the disk under the influence of its own internal shear dynamo fields, (e.g. Balbus & Hawley 1998, Balbus 2003). At the inner disk radius the magnetic field of the central MECO is much stronger than the shear dynamo field generated within the inner accretion disk.

The various spectral states begin with a true quiescence at luminosity L_q , before the outburst begins. There is usually only a factor of a few change of luminosity by the time the accretion disk engages the magnetic field of the central object (MECO, NS or WD) at the “light cylinder”. This marks the maximum quiescent luminosity state at $L_{q,max}$. This is followed by a low/hard spectral state as the outer disk encroaches into the magnetic field. From the light cylinder radius to the “corotation radius”, r_c , the x-ray luminosity of the accretion disk may increase by a factor of $\sim 10^3 - 10^6$. At the “corotation radius”, r_c , the Keplerian orbit frequency of the inner disk matches the spin frequency of the central object and the propeller effect ceases; at least until a significant speed differential between the inner disk and the central object magnetic field is re-established. When the inner disk penetrates inside corotation, large fractions of the accreting plasma can continue on to the central object and produce a spectral state switch to softer and brighter emissions.

A magnetic propeller regime (Ilarianov & Sunyaev 1975, Stella, White & Rosner 1986, Cui 1997, Zhang, Yu & Zhang 1998, Campana et al. 1998, 2002) exists until the inner disk pushes inside the co-rotation radius, r_c . Plasma may depart in a jet, or as an outflow back over the disk as it is accelerated on outwardly curved or open magnetic field lines. Radio images of both flows have been seen (Paragi et al. 2002). Equatorial outflows may contribute to the low state hard spectrum by bulk Comptonization of soft photons in the outflow, however, it seems likely that the hard spectrum originates primarily in patchy coronal flares (Merloni & Fabian 2002) on a conventional geometrically thin, optically thick accretion disk. Both outflow comptonization and coronal flares are compatible with partial covering models for dipping sources, in which the hard spectral region seems to be extended (Church 2001, Church & Balucinska-Church 2001). Alternatively, a compact jet (Corbel & Fender 2002) might be a major contributor to the hard spectrum, but if so, its

x-ray luminosity must fortuitously match the power that would be dissipated in a conventional thin disk. The power law emissions of the low/hard state are usually cut off below ~ 100 keV, consistent with a coronal temperature of $20 - 50$ keV. Bulk comptonization would be expected to produce higher energies.

The inner disk reaching the corotation radius marks the end of the low/hard state at its maximum luminosity, L_c . If there is sufficient accretion disk pressure to push the magnetosphere inward, intermediate or high/soft states may be produced as the inner disk continues to fill in to the innermost marginally stable orbit ($5.24R_g$ for an exponential metric MECO). Inside the marginally stable orbit plasma accelerates in a supersonic flow on toward the central object.

In previous work (RL02, RL03, RL04, RL06), corotation radii in the range $30 - 70R_g$ have been found. Since the accretion disk can be adequately described by Newtonian gravity to radii this small, it doesn't matter which metric we might use to describe the outer accretion disk of a MECO. Even at the innermost marginally stable orbit the differences between exponential and Schwarzschild metrics are small enough to be disregarded; however, Newtonian gravity would predict too much energy release inside the corotation radius because it fails to account for relativistic mass increase. With these caveats, there is no need to recapitulate the evidence previously analyzed by Robertson & Leiter. The equations that they used in their analyses are described below and recapitulated in Table 1.

5 Characteristic Disk Luminosity Relations

Using units of $10^{27} G cm^3$ for magnetic moments, μ , 100 Hz for spin, ν , 10^6 cm for radii, r , 10^{15} g/s for accretion rates, \dot{m} , solar mass units, m , the magnetosphere radius was found (RL04) to be:

$$r_m = 8 \times 10^6 \left(\frac{\mu_{27}^4}{m \dot{m}_{15}^2} \right)^{1/7} cm \quad (25)$$

and the disk luminosity is

$$L = \frac{GM\dot{m}}{2r_m} \quad (26)$$

The co-rotation radius, at which disk Keplerian and magnetosphere spins match is:

$$r_c = 7 \times 10^6 \left(\frac{m}{\nu_2^2} \right)^{1/3} cm \quad (27)$$

The low state disk luminosity at the co-rotation radius is the maximum luminosity of the true low state and is given by:

$$L_c = \frac{GM\dot{m}}{2r_c} = 1.5 \times 10^{34} \mu_{27}^2 \nu_2^3 m^{-1} erg/s \quad (28)$$

The minimum high state luminosity for all accreting matter being able to reach the central object occurs at approximately the same accretion rate as for L_c and is given by:

$$L_{min} = \xi \dot{m} c^2 = 1.4 \times 10^{36} \xi \mu_{27}^2 \nu_2^{7/3} m^{-5/3} \text{ erg/s} \quad (29)$$

Where $\xi \sim 0.055$ for MECO at the innermost marginally stable orbit. The supersonic flow beyond that should generate little luminosity and, as noted above, the central MECO would produce very little. For neutrons stars, $\xi \sim 0.14$ for accretion reaching the star surface.

In true quiescence, the inner disk radius is larger than the light cylinder radius. In NS and GBHC, the inner disk may be ablated due to radiation from the central object. The inner disk radius can be ablated to distances larger than $5 \times 10^4 \text{ km}$ because optically thick material can be heated to $\sim 5000K$ and ionized by the radiation. The maximum disk luminosity of the true quiescent state occurs with the inner disk radius at the light cylinder, $r_{lc} = c/\omega_s = r_m$. The maximum luminosity of the quiescent state is typically a factor of a few larger than the average observed quiescent luminosity.

$$L_{q,max} = (2.7 \times 10^{30} \text{ erg/s}) \mu_{27}^2 \nu_2^{9/2} m^{1/2} \quad (30)$$

The luminosity of the true quiescent state was calculated (RL03, RL06) from the correlation of spin-down energy loss rate (Possenti et al 2002) with x-ray luminosity in the soft x-ray band from $\sim 0.5 - 10 \text{ keV}$, assuming that the luminosity is that of an aligned spinning magnetic dipole.

$$L_q = \beta \times \frac{32\pi^4 \mu^2 \nu^4}{3c^3} = 3.8 \times 10^{33} \beta \mu_{27}^2 \nu_2^4 \text{ erg/s} \quad (31)$$

For MECO-GBHC $\beta \sim 3 \times 10^{-4}$ was found (RL06).

Since the magnetic moment, μ_{27} , enters each of the above luminosity equations it can be eliminated from ratios of these luminosities, leaving relations involving only masses and spins. For known masses, the ratios then yield the spins. Alternatively, if the spin is known from burst oscillations, pulses or spectral fit determinations of r_c , one only needs one measured luminosity, L_c or L_{min} at the end of the transition into the soft state, to enable calculation of the remaining μ_{27} and L_q (see RL02, RL06).

For GBHC, it is a common finding that the low state inner disk radius is much larger than that of the marginally stable accretion disk orbit; e.g. (Markoff, Falcke & Fender 2001, Życki, Done & Smith 1997a,b, Done & Życki 1999, Wilson & Done 2001). The presence of a magnetosphere is an obvious explanation. An empty inner disk region has also been found from observations of microlensing of the quasars Q0957+561 and Q2237+0305 (Schild, Leiter & Robertson 2006, 2008). These observed empty inner disk structures are consistent with the strongly magnetic MECO model but do not accord with standard thin disk models of accretion flows into a black hole. Finally, note that if an inner disk radius is found for

a known mass from fitting disk spectra for conditions near the low/high spectral state transition, the MECO spin frequency follows from the classical Kepler relation $\nu_s = 2\pi\nu_s = \sqrt{GM/r^3}$.

To summarize, the magnetosphere/disk interaction affects nearly all of the spectral characteristics of NS and GBHC in LMXB and dwarf nova systems and accounts for them in a unified and complete way, including jet formation and radio emissions. This model is solidly consistent with accreting NS systems, for which intrinsic magnetic moments obtained from spin-down measurements allow little choice. Even their relatively weak magnetic fields are too strong to ignore. Since the similar characteristics of GBHC are cleanly explained by the same model, the MECO offers a unified theory of LMXB phenomenology as well as extensions to AGN. Since MECO lifetimes are orders of magnitude greater than a Hubble time, they provide an elegant and unified framework for understanding the broad range of observations associated with GBHC and AGN.

6 Low State Mass Ejection and Radio Emission

The radio flux, F_ν , of jet sources has a power law dependence on frequency of the form

$$F_\nu \propto \nu^{-\alpha} \quad (32)$$

It is believed to originate in jet outflows and has been shown to be correlated with the low state x-ray luminosity [Merloni, Heinz & DiMatteo 2003], with $F_\nu \sim L_x^{0.7}$. The radio luminosity of a jet is a function of the rate at which the magnetosphere can do work on the inner ring of the disk. This depends on the relative speed between the magnetosphere and the inner disk; i.e., $\dot{E} = \tau(\omega_s - \omega_k)$, or (RL04)

$$\dot{E} = 0.015 \frac{\mu^2 \omega_s (1 - \frac{\omega_k}{\omega_s})}{r^3} \propto \mu^2 M^{-3} \dot{m}_{Edd}^{6/7} \omega_s (1 - \frac{\omega_k}{\omega_s}) \quad (33)$$

Here \dot{m}_{Edd} is the mass accretion rate divided by the rate that would produce luminosity at the Eddington limit for mass M .

Disk mass, spiraling in quasi-Keplerian orbits from negligible speed at radial infinity must regain at least as much energy as was radiated away in order to escape. For this to be provided by the magnetosphere requires $\dot{E} \geq GM\dot{M}/2r$, from which $\omega_k \leq 2\omega_s/3$. Thus the magnetosphere alone is incapable of completely ejecting all of the accreting matter once the inner disk reaches this limit and the radio luminosity will be commensurately reduced and ultimately cut off at maximum x-ray luminosity for the low state and $\omega_k = \omega_s$. For very rapid inner disk transit through the corotation radius, fast relative motion between inner disk and magnetosphere can heat the inner disk plasma and strong bursts of radiation pressure from either the inner disk or the central object may help to drive large outflows while an extended jet

Table 1: MECO Model Equations

MECO Physical Quantity	Equation	Scaling
1. Surface Redshift - (RL06 2)	$1 + z_s = 5.67 \times 10^7 m^{1/2}$	$m^{1/2}$
2. Quiescent Surface Luminosity L_∞ - (RL06 29)	$L_\infty = 1.26 \times 10^{38} m / (1 + z_s) \text{ erg s}^{-1}$	$m^{1/2}$
3. Quiescent Surface Temp T_∞ - (Eq 12)	$T_\infty = 2 \times 10^6 / [m(1 + z_s)]^{1/4} = 2.3 \times 10^4 m^{-3/8} \text{ K}$	$m^{-3/8}$
4. Photosphere Temp. T_p	$T_p = 2.5 \times 10^8 m^{-0.034} \text{ K}$	$m^{-0.034}$
5. Photosphere redshift z_p	$1 + z_p = 11000 m^{0.343}$	$m^{0.343}$
6. GBHC Rotation Rate, units 10^2 Hz - (RL06 47)	$\nu_2 = 0.89 [L_{q,32}/m]^{0.763} / L_{c,36} \approx 0.6 s_1 / m \text{ Hz}^a$	m^{-1}
7. GBHC Quiescent Lum., units $10^{32} \text{ erg s}^{-1}$ - (RL06 45, 46)	$L_{q,32} = 1.17 [\nu_2 L_{c,36}]^{1.31} = 4.8 \times 10^{-3} \mu_{27}^{2.62} \nu_2^{5.24} m^{-0.31} \text{ erg s}^{-1}$	m
8. Co-rotation Radius - (RL06 40)	$R_c = 7 \times 10^6 [m/\nu_2^2]^{1/3} \text{ cm}$	m
9. Low State Luminosity at R_c , units $10^{36} \text{ erg s}^{-1}$ (RL06 41)	$L_{c,36} = 0.015 \mu_{27,\infty}^2 \nu_2^3 / m \text{ erg s}^{-1}$	m
10. Magnetic Moment, units 10^{27} G cm^3 - (RL06 41, 47)	$\mu_{27,\infty} = 8.16 [L_{c,36} m / \nu_2^3]^{1/2} \mu = 1.7 \times 10^{28} m^{5/2} \text{ G cm}^3$	$m^{5/2}$
11. Disk Accr. Magnetosphere Radius - (RL06 38)	$r_m(\text{disc}) = 8 \times 10^6 [\mu_{27,\infty}^4 / (m \dot{m}_{15}^2)]^{1/7} \text{ cm}$	m
12. Spherical Accr. Magnetosphere Radius	$r_m(\text{sphere})$ or axial $z_m(\text{in}) = 2.3 \times 10^7 [\mu_{27,\infty}^4 / (m \dot{m}_{15}^2)]^{1/7} \text{ cm}$	m
13. Spher. Accr. Eq. Mag. Rad. Rotating Dipole (RTTL03)	$r_m(\text{out}) = 1.2 \times 10^7 [\mu_{27}^2 / (\dot{m}_{15} \nu_2)]^{1/5} \text{ cm}$	m
14. Equator Poloid. Mag. Field - (RL06 41, 47, $B_\perp = \mu_\infty / r^3$)	$B_{\infty,10} = 250 m^{-3} (R_g/r)^3 {}^b [L_{c,36} / (m^5 \nu_2^3)]^{1/2} \text{ gauss}$	$m^{-1/2}$
15. Low State Jet Radio Luminosity - (RL04 18, 19)	$L_{\text{radio},36} = 10^{-6.64} m^{0.84} L_{x,36}^{2/3} [1 - (L_{x,36}/L_{c,36})^{1/3}] \text{ erg s}^{-1}$	$m^{3/2}$

a: s_1 is a small dimensionless numerical factor ($s_1 \sim 1$ for GBHC, see text)

b: $R_g = GM/c^2$

structure is still largely intact. This process has been calculated³ using pressures and poloidal magnetic fields of unspecified origins. A MECO is obviously capable of supplying both the field and disk radiation pressure. The hysteresis of the low/high and high/low state transitions may be associated with the need for the inner disk to be completely beyond the corotation radius before a jet can be regenerated after it has subsided.

For the convenience of readers who might wish to relate MECO properties to observations of compact astrophysical objects, a number of useful relations are given in Table 1. Many of the parameters are given in terms of quiescent x-ray luminosity L_q , or the luminosity, L_c , at the transition high/soft \rightarrow low/hard state since these are often measurable quantities. Mass scaling relationships for MECO are listed in the right hand column of Table 1. They have been shown to account for the quasar accretion structures (Schild, Leiter & Robertson 2006, 2008) revealed by microlensing observations of the quasars Q0957+561 and Q2237+0305. The observed structures are consistent with the strongly magnetic MECO model but do not accord with standard thin disk models of accretion flows into a black hole. Since even the nearest GBHC are much too small to be resolved in the detail shown by the microlensing techniques which were used in the study of Q0957 and Q2237, Sgr A* is likely the only remaining black hole candidate for which resolved images might reveal whether or not it possesses a magnetic moment. For this reason it is important that it be tested.

³though for inner disk radii inside the marginally stable orbit [Chou & Tajima 1999]. This would be a high state jet, but the basic mechanism would work in low states given strong enough magnetic fields.

7 Discussion and Summary

An enormous body of physics scholarship developed primarily over the last half century has been built on the assumption that trapped surfaces leading to event horizons and curvature singularities can actually exist in nature. Misner, Thorne & Wheeler [1973], for example in Sec. 34.6, clearly state that this is an assumption that underlies the well-known singularity theorems of Hawking and Penrose. Unfortunately, this assumption has been elevated to the status of an accepted fact without proof. Now we are finding that the idea of an event horizon presents problems for the structure of theoretical physics (e.g., Abramowicz, Kluzniak & Lasota 2014, Hawking 2014). While previous successful ECO-MECO models have shown that giving up the idea of event horizons does not run afoul of any astrophysical observations, problems associated with their occurrence within gravity theory itself may remain.

If gravity theory needs to be altered to remove the possible occurrence of event horizons, the exponential function for gravitational redshifts as developed above (see also Appendices A & B) would seem to be a good place to start. It is simple and it rests on accepted physical principles. Regardless of anything else that might be needed for the structure of theoretical physics, nothing else is needed for a quantitative accounting of the observed properties of GBHC, AGN, neutron stars (see Appendix C). Even cosmological redshifts without “dark energy” (Robertson 2015) can be accommodated by allowing for time dependence in the exponential metric. As shown here, the MECO model based on the exponential metric differs in only minor details from the previous version that was based on a Schwarzschild metric, but it has the virtue of resting entirely upon a metric that is incapable of producing an event horizon.

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APPENDIX

A Einstein's elevators

Consider two elevators, one at rest on a planet where the local gravitational free-fall acceleration would be g . Let the other be out in free space away from gravitational fields. Let it be equipped with a rocket engine and accelerating at the same rate, g , in the z direction as determined by its on-board accelerometer. At the time the second elevator begins to accelerate, let a photon be emitted from a source at its floor and let it be absorbed later in a detector in its ceiling, a distance L away in the frame of the elevator. While the photon is in transit, the detector acquires some speed, v , relative to inertial frames. From the position of the detector, it is the same as if the source were receding from it at speed v . Thus if the frequency of the photon emitted at the floor is ν_0 , the detector will detect the Doppler shifted frequency

$$\nu = \frac{\nu_0(1 - v/c)}{\sqrt{1 - v^2/c^2}} \quad (34)$$

We can determine the speed, v , of the ceiling photon detector from the special relativistic relation

$$a_z = \frac{dv}{dt} = \frac{a'_z}{(\gamma^3)(1 + u'_z v/c^2)^3} = \frac{g}{\gamma^3} \quad (35)$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ and $u'_z = 0$ is the detector speed relative to an inertial frame that is comoving and coincident at the time the photon reaches the detector. Time increments, dT in the elevator are contracted such that $dT = dt/\gamma$. Substituting into Eq. 35, integrating and setting $T = L/c$, we obtain

$$v/c = \tanh(gL/c^2) \quad (36)$$

Substituting Eq. A3 into Eq. A1, there follows

$$\nu = \nu_0 e^{-gL/c^2} \quad (37)$$

By the principle of equivalence an elevator at rest in an equivalent gravitational field, would have to produce the same frequency shift gravitationally. In this elevator, the change of (dimensionless) gravitational potential of the ceiling relative to the floor is, of course, $u(z = L) = gL/c^2$. So the red shift as photons move upward from the source is given by $1 + z = e^{u(z)}$. It is necessary to require **EXACT** adherence to this exponential form because the acceleration might be large enough to make the accelerated elevator reach a relativistic speed before the photon arrives at the ceiling detector. This photon red shift result was derived by Einstein in a 1907 paper (Schwartz 1977). For a time after 1907, Einstein maintained that the metric coefficient g_{00} must be a strictly exponential function of gravitational potential, but his final development of general relativity fails to satisfy the requirement. The Schwarzschild metric only agrees with the requirement to first order.

B Exponential Metric and General Relativity

The (dimensionless) potential at distance r from a mass, M , is $u(r) = GM/c^2 r$. In the exponential metric used here,

$$ds^2 = e^{-2u(r)} c^2 dt^2 - e^{2u(r)} (dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2) \quad (38)$$

the gravitational field energy density would be a tensor, t_i^j with components

$$t_0^0 = -t_1^1 = t_2^2 = t_3^3 = \frac{-c^4}{8\pi G} [e^{-u} \partial_r u(r)]^2 \quad (39)$$

If Einstein's gravitational field equation is modified to include this field stress-energy tensor as a source term in the right member the field equations would become (Yilmaz 1971)

$$G_i^j = -(8\pi G/c^4)(T_i^j + t_i^j) \quad (40)$$

In the space exterior to M , where $T_i^j = 0$, the G_0^0 equation reduces to $\nabla^2 u(r) = 0$, for which $u(r) = GM/c^2 r$ is, indeed, a solution and the G_1^1, G_2^2 , and G_3^3 equations are satisfied.

Yilmaz theory has been developed further (e.g., Yilmaz 1975, 1977, 1980, 1992). As a theory of particles and fields it seems to be a viable alternative to General Relativity, however, it needs clarification of some issues concerning the field equations and the form of t_i^j in a matter continuum. For a uniform cosmological distribution of dust particles it was recently shown (Robertson 2015) to correctly account for the observed cosmological redshift-luminosity relation of SNe 1a.

C Neutron Star Mass

With nothing more than a plausible equation for the potential in the interior of a mass distribution, a hydrostatic equilibrium equation and an equation of state for neutrons, it is possible to show that there should be a limiting mass for neutron stars that is well below the mass of any known GBHC. The potential equation is assumed to be

$$e^{-2u} \nabla^2 u = -\frac{8\pi G}{c^4} (\rho c^2 + 3p) \quad (41)$$

where ρ is mass-energy density, p is the pressure, and $3p$ its contribution to active gravitational mass density. The hydrostatic pressure equation consistent with g_{00} from the exponential metric would be (Weinberg 1972, p127)

$$p' = u'(\rho c^2 + p) \quad (42)$$

Where primes denote derivatives with respect to r . What is needed for the solution of these equations is a neutron equation of state and an initial trial central pressure $p(0)$. For the latter, it was assumed that $p(0) = \rho(0)c^2/3$ corresponds to the

maximum realistic pressure because the core would be fully relativistic under these conditions and no longer cool enough to permit the neglect of radiation.

The AV14+UVII model equation of state of Wiringa, Fiks, and Fabrocini was used here (Wiringa, Fiks & Fabrocini 1988). Densities from their Table VI were fitted to a quartic polynomial in $p^{1/3}$. The quartic polynomial for neutron densities was of the form $\rho = \sum_n a_n (p^{1/3})^n$. The coefficients are: $a_0 = -0.9167367$, $a_1 = 6.960282$, $a_2 = -1.462936$, $a_3 = 0.267646$, $a_4 = -0.0112676$. Fitting errors were below one percent over the range of densities used in these calculations. The initial pressure and density corresponding to $p(0) = \rho(0)c^2/3$ were $p(0) = 47.7 \times 10^{34} \text{ erg cm}^{-3}$ and $\rho(0) = 15.9 \times 10^{14} \text{ g cm}^{-3}$. Using these values, a maximum neutron star mass of $M = 1.57M_\odot$ and surface radius $R = re^u = 9.7 \text{ km}$ was found. For a canonical $1.4M_\odot$ star, the central pressure and density were found to be $25.9 \times 10^{34} \text{ erg cm}^{-3}$, $12.9 \times 10^{14} \text{ g cm}^{-3}$ with a surface radius of 12.9 km .

After selection of an initial central density and corresponding pressure, the solution proceeded in steps outward with corresponding decrements of pressure until reaching $p = 0$. At that point the numerical solution was forced to match the external potential and its derivative. Initial conditions used were $u(0) = u'(0) = 0$, $p'(0) = 0$. As a check on the validity of using the polynomial, the field equations of the Schwarzschild metric were solved numerically in the same stepwise fashion. Stellar radii and masses agreed to better than one percent with those shown in Table VI of Wiringa, Fiks & Fabrocini. Since their values were calculated via the Tolman, Oppenheimer, Volkoff equation which was derived for the Schwarzschild metric, this should be expected; however, a failure to agree would have indicated a problem with the numerical methods used here.

D The $\gamma + \gamma \leftrightarrow e^\pm$ Phase Transition

It is well-known that a spherical volume of radius R containing a luminosity L_γ of gamma ray photons with energies $> 1 \text{ MeV}$,⁴ will become optically thick to the $\gamma + \gamma \leftrightarrow e^\pm$ process when the optical depth is

$$\tau_\pm \sim n_\gamma \sigma_{\gamma\gamma} R \sim 1 \quad (43)$$

and

$$n_\gamma \sim L_\gamma / (4\pi R^2 m_e c^3) \quad (44)$$

is the number density of γ -ray photons with energies $\sim 1 \text{ MeV}$, L_γ is the *gamma*-ray luminosity, R is the radius of the volume, and $\sigma_{\gamma\gamma}$ is the pair production cross section.

Since $\sigma_{\gamma\gamma} \sim \sigma_T$, the Thompson cross-section, near threshold, it follows that the system becomes optically thick to photon-photon pair production when the

⁴1 MeV photons correspond to $T \sim 10^{10} \text{ K}$, which is only slightly beyond the pair threshold, and easily within reach in gravitational collapse.

numerical value of its compactness parameter, L_γ/R , is

$$L_\gamma/R \sim 4\pi m_e c^3 / \sigma_T \sim 5 \times 10^{29} \text{ erg cm}^{-1} \text{ sec}^{-1} \quad (45)$$

Hence $\tau_\pm > 1$ will be satisfied for systems with compactness

$$L_\gamma/R \approx 10^{30} \text{ erg cm}^{-1} \text{ sec}^{-1} \quad (46)$$

Using $R \sim R_g/u(R) = R_g/\ln(1+z)$, taking the proper length and volume into consideration, and substituting from Eq. 6 for L_γ , the proper optical depth to photon-photon pair production has the very large value

$$\tau_\pm(1+z) \sim (1+z)(L_{\gamma,30}/R) \sim 850(1+z)^2 \ln(1+z) \gg 1 \text{ erg cm}^{-1} \text{ sec}^{-1} \quad (47)$$

Thus the resultant $\gamma + \gamma \leftrightarrow e^\pm$ phase transition in the MECO surface magnetic field, B_S , creates an optically thick pair dominated plasma. Taking the photon escape cone factor $\sim 1/(1+z)^2$ into account, the process generates a net outward non-polytropic radiation pressure

$$P \propto \ln(1+z) m B_S^4 \quad (48)$$

on the MECO surface that can greatly exceed the thermal radiation pressure that would correspond to a pair production threshold temperature of $6 \times 10^9 \text{ K}$. The hydrostatic equilibrium is mass scale invariantly stabilized at the threshold of magnetically produced pairs at the baryon surface.